Polarization issues – a tutorial

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Topics

- Linear vs. circular polarization feeds
  - Which is better?
  - Do we have a choice?
- Cross-polarization
  - Effects in geodetic VLBI
  - Measuring cross-polarization
  - Correcting for cross-polarization
- Using dual-linear feeds in geodetic VLBI

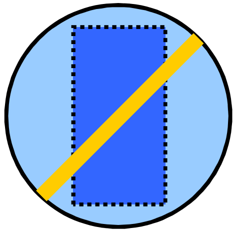
Linear vs. circular polarization feeds

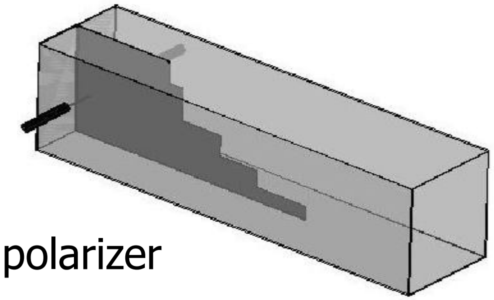
- Which is better?
  - Advantages of linear:
    - Easier to build
  - Advantages of circular:
    - Fringe amplitude is insensitive to feed rotation.
      - Fringe benefit: Can do geodetic VLBI with a single sense of circular polarization (RCP or LCP).
      - But VLBI2010 needs the extra SNR from dual polarization anyway.
    - Polarization structure of radio sources is easier to map with circular feeds.
- Do we have a choice? Yes, probably, maybe…..

Circular pol feeds at centimeter wavelengths

- Most circular polarizers are intrinsically narrowband (<2:1 bandwidth).
- Examples:
  - Dielectric slab in circular waveguide ahead of rectangular waveguide – E-field parallel to slab is slowed relative to perpendicular component
  - Septum polarizer in square waveguide – phase velocities differ for E-field components perpendicular and parallel to septum
- There are no suitable (low loss, well-behaved beam pattern) circular-pol feeds operating over 2-14 GHz, but quadrature LNA/receivers can give circular.

dielectric in circular waveguide

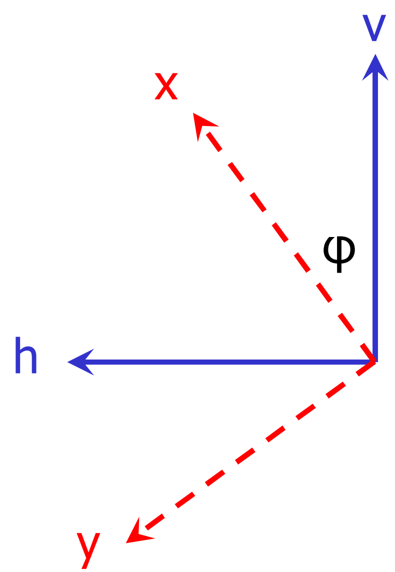




septum polarizer

Feed rotation with linear feeds

- For az-el antenna mount, feed orientation (x,y) on sky is not fixed but varies as parallactic angle φ, which depends on hour angle, source declination, and antenna latitude.
- Feed orientation also varies for an x-y antenna mount.
- For polar antenna mount, feed orientation relative to source is fixed. But we're not going to use polar mounts!!



(v,h) = (N-S,E-W) source E-field components
(x,y) = feed dipole orientation

Linear feeds in VLBI

- The feed rotation angles for two widely separated antennas can differ by -180° to +180°.
  - Large differences can also occur for antennas <1000 km apart if the source passes close to the zenith at one site.
- If the feed angles differ by 90°, the x dipoles at the two sites will pick up orthogonal polarizations from the source.
- Because the emission from extragalactic sources is incoherent, the XX or YY correlation for a 90° difference will be zero.
- Ionospheric Faraday rotation can cause the polarization plane from the source to be rotated at the station by up to ~10° at 2 GHz.

Options for using linear feeds in VLBI

- Use a feed sensitive to only one polarization component (x or y), and rotate it axially to maintain a fixed orientation on the sky. Issues:
  - Does not account for ionospheric Faraday rotation.
  - Want extra SNR from two polarizations for VLBI2010.
  - Dual-polarization is needed to measure cross-pol.
- Use a feed sensitive to both polarization components.
  - Generate circular polarization at the station after digitization.
    - Normally need to correlate only RR and LL.
  - Correlate all parallel/cross-hands XX, YY, XY, YX.
    - Increased workload at correlator.

Cross-polarization with circular feeds

- For a nominally RCP feed, its output voltage V_R depends on the incident RCP and LCP E-fields as

$$V_R = G_R\,(E_R e^{-i\phi} + D_R\,E_L\,e^{i\phi})$$

where D_R gives the "leakage" from the LCP field.

- The cross-correlation for two RCP feeds, relative to that for two perfect feeds, on a source with no net CP is

$$1 + D_{R1}\,D^*_{R2}\,e^{2i\Delta} + D_{R1}\frac{Q-iU}{I}\,e^{2i\phi_1} + D^*_{R2}\frac{Q+iU}{I}\,e^{-2i\phi_2}$$

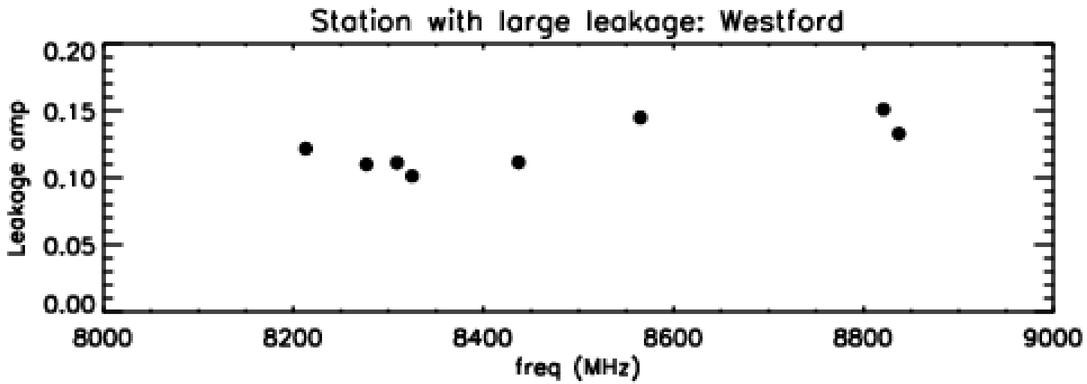
where (I,Q,U,V) are the Stokes parameters:
  - I = total intensity
  - Q,U = linearly polarized intensity
  - V = circularly polarized intensity

Errors from cross-polarization in circular feeds

- The leakage terms cause the correlation to be sensitive to feed orientation. Errors that vary with time and geometry are thereby introduced into the geodetic phase and delay.
- Errors from (leakage) x (source linear pol):
  - Source linear polarization is typically a few percent and no more than 10%.
  - Typical S/X D terms are 0.05-0.15 (-26 to -16 dB).
  - For 10% polarization and -20 dB D term:
    - Phase error up to 0.01 radian
    - Delay error over 500 MHz up to 3 ps
- Error from LCP leakage in two feeds is comparable to above.
  - Phase/delay errors around a station triangle do not close.

Measuring cross-polarization in circular feeds

- Special experiments to measure cross-pol in S/X feeds have been conducted.
- The most recent, and by far the most complete, was done by Alessandra Bertarini (Bonn).
  - 10 dual-pol VLBA stations (need dual-pol to measure LR and RL cross-corr.) and 10 S/X geodetic stations.
  - Data have been correlated and are being analyzed. Sample results (for more, come to Bordeaux!):



Correcting for circular feed cross-polarization

- Once the D terms have been measured, the geodetic delays can be corrected for the LCP-LCP leakage.
- Correction for the (leakage) x (source linear pol) errors requires detailed knowledge of the polarization structure of the sources.
  - Such source maps are available only in extremely rare cases at present.
  - The situation may change with VLBI2010.
- If the D terms are small enough, correction is unnecessary.

Cross-polarization with linear feeds

- For an imperfect linear feed, the output voltages *(x′,y′)* include contributions from the two pol components:

  $x' = x + D_x\, y$ and $y' = y + D_y\, x$

- The four correlation cross-products are

$$
\begin{aligned}
2\,\langle x_1' x_2'^{*} \rangle \,/\, G_{x1} G_{x2}^{*} &= +I\cos\Delta + Q\cos\Sigma + U\sin\Sigma - iV\sin\Delta + I\,(-D_{x1} + D_{x2}^{*})\sin\Delta \\
2\,\langle y_1' y_2'^{*} \rangle \,/\, G_{y1} G_{y2}^{*} &= +I\cos\Delta - Q\cos\Sigma - U\sin\Sigma - iV\sin\Delta + I\,(+D_{y1} - D_{y2}^{*})\sin\Delta \\
2\,\langle x_1' y_2'^{*} \rangle \,/\, G_{x1} G_{y2}^{*} &= +I\sin\Delta - Q\sin\Sigma + U\cos\Sigma + iV\cos\Delta + I\,(+D_{x1} + D_{y2}^{*})\cos\Delta \\
2\,\langle y_1' x_2'^{*} \rangle \,/\, G_{y1} G_{x2}^{*} &= -I\sin\Delta - Q\sin\Sigma + U\cos\Sigma - iV\cos\Delta + I\,(+D_{y1} + D_{x2}^{*})\cos\Delta
\end{aligned}
$$

where Δ = feed angle difference, Σ = their sum, and only the highest-order terms involving D are retained.

- Note the expected dependence of the dominant I (total intensity) term in the XX and YY products on cos Δ.

Measuring cross polarization in linear feeds

- It is anticipated that, as with circular feeds, the linear-pol *D* terms can be measured in dual-pol VLBI observations, but it has never been done!
  - Centimeter-wavelength VLBI antennas have almost universally employed circular feeds to date.
  - Some connected-element interferometers use linear feeds, but their Δ's are always very close to zero.
- It may be possible to measure cross-pol directly with a far-field (14 km at 15 GHz for a 12-m dish) transmitter.
- The effect on the delays depends on how the delay observable is constructed.
- D terms for linear feeds are usually smaller than for circular. → The cross-pol errors may be negligible.

Using dual-linear feeds in geodetic VLBI

- Recap: Two options for avoiding the problem with fringe amplitude nulls as the feed rotation angles vary:
  - Make circular from linear in the station digital signals.
  - Correlate linears and then sum the four cross-products as if circular signals had been correlated.
- Get the same results either way:

$$
\begin{aligned}
RR &\equiv \langle (x_1'/G_{x1} + iy_1'/G_{y1})\,(x_2'/G_{x2} + iy_2'/G_{y2})^* \rangle \\
&= [\,(I+V) + \frac{i}{2} I(-D_{x1} + D_{y1} + D_{x2}^* - D_{y2}^*)\,]\, e^{-i\Delta} \\
LL &\equiv \langle (x_1'/G_{x1} - iy_1'/G_{y1})\,(x_2'/G_{x2} - iy_2'/G_{y2})^* \rangle \\
&= [\,(I-V) - \frac{i}{2} I(-D_{x1} + D_{y1} + D_{x2}^* - D_{y2}^*)\,]\, e^{+i\Delta}
\end{aligned}
$$

- Note that the magnitude is independent of feed rotation.

Leakage error in *RR* and *LL* from linears

- For a 20-dB D term, each of the leakage terms in *RR* and *LL* could introduce a 0.05 radian phase error, or up to 16 ps delay error.
- But the feed angle dependence of the leakage contributions is identical to that of the desired cross-product that is proportional to I ± V.
  - The D terms should be constant over hours to years.
  - Any systematic delay error they cause will be similarly constant and so will be "soaked up" in the geodetic solution as a clock error.
- In general, D terms are smaller for linear feeds than for circular.

Constructing circulars from linears

- Whether the circular signals are constructed at the station or at the correlator, the relative complex gains (amplitude and phase) between the two linear signals must be either measured ahead of time or determined from the data.
- Noise and/or phase cal signals radiated into the feed from either a circular-pol antenna or a linear dipole oriented midway between the two arms of the feed could provide calibration signals to measure the relative gains.
- Fringe amplitudes on weakly polarized sources can give the relative gain magnitude.
- With strong sources yielding high SNR in all four cross-products, "manual phases" can be determined to align the four visibility phasors. With phase cal providing the phase frequency dependence, the delay can then be calculated.