Rotational Motions in Seismology: Theory, Instruments, Observations

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What is rotation (in seismology)?

Rotations from seismic arrays?
- Observations with a "finite difference array"
- Bam, 26 December, 2003, M6.8

What effects can we expect?
- Homogeneous
- Heterogeneous
- Earthquake scenarios

The M8.3 Hokkaido earthquake: Observations and modelling

Conclusions and future
Damage due to rotations
What is rotation?

The motion of a (non-deformable) body is uniquely specified by three components of displacement (determined by a classical seismometer) and three components of rotation.

It is standard to observe translational motions but the study of rotations had little attention as the effects generated by earthquakes are thought to be small (e.g. Bouchon and Aki, 1982).

Recently there has been observational evidence that rotational motions may indeed be strong (e.g. Takeo and Ito, 1997; Takeo, 1998)
Let's ask Aki ...

„The state-of-the-art sensitivity of the general rotation-sensor is not yet enough for a useful geophysical application“ (Aki and Richards, Quantitative Seismology, 1980)

„... note the utility of measuring rotation near a rupturing fault plane (...), but as of this writing seismology still awaits a suitable instrument for making such measurements“ (Aki and Richards, Quantitative Seismology, 2nd edition 2002)
Linear Elasticity

The partial derivatives of the vector components

\[ \frac{\partial u_i}{\partial x_k} \]

represent a second-rank tensor which can be resolved into a symmetric and anti-symmetric part:

\[
\delta u_i = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \delta x_k - \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} - \frac{\partial u_i}{\partial x_k} \right) \delta x_k
\]

- symmetric
- deformation
- antisymmetric
- pure rotation
Rotation is the **curl** of the wavefield

\[
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
= \nabla \times \mathbf{v} =
\begin{pmatrix}
\partial_y v_z - \partial_z v_y \\
\partial_z v_x - \partial_x v_z \\
\partial_x v_y - \partial_y v_x
\end{pmatrix}
\]

Rotation

**Rotation sensor**

Velocity

**Seismometer**
Rotation from translations?

\[ \omega_z = \partial_x v_y - \partial_y v_x \]

Rotational measurement with seismometers
The Finite-Difference Array Experiment
Field work
Results from Bam earthquake, 26 Dec 2003, M6.8
Results from Bam earthquake, 26 Dec 2003, M6.8
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Radiation from a **double-couple** point source

Geometry we use to express the seismic wavefield radiated by point double-couple source with area $A$ and slip $\Delta u$

Here the fault plane is the $x_1x_2$-plane and the slip is in $x_1$-direction.

**FIGURE 5** Cartesian and polar coordinate systems for analysis of radiation by a slip patch with area $A$ and average slip $\langle \Delta u(t) \rangle$. 
Radiation from a point source

\[
u(x, t) = \frac{1}{4\pi\rho} A^N \frac{1}{r^4} \int_{r/v_p}^{r/v_s} \tau M_0(t - \tau) d\tau
+ \frac{1}{4\pi\rho v_p^2} A^{IP} \frac{1}{r^2} M_0(t - r/v_p)
+ \frac{1}{4\pi\rho v_s^2} A^{IS} \frac{1}{r^2} M_0(t - r/v_s)
+ \frac{1}{4\pi\rho v_p^3} A^{FP} \frac{1}{r} \dot{M}_0(t - r/v_p)
+ \frac{1}{4\pi\rho v_s^3} A^{FS} \frac{1}{r} \ddot{M}_0(t - r/v_s).
\]

- \(u\): ground displacement as a function of space and time
- \(\rho\): density
- \(r\): distance from source
- \(V_s\): shear velocity
- \(V_p\): P-velocity
- \(N\): near field
- IP/S: intermediate field
- FP/S: far field
- \(M_0\): seismic moment

\[
\begin{align*}
A^N &= 9 \sin 2\theta \cos \phi \dot{r} - 6 (\cos 2\theta \cos \phi \dot{\theta} - \cos \theta \sin \phi \dot{\phi}), \\
A^{IP} &= 4 \sin 2\theta \cos \phi \dot{r} - 2 (\cos 2\theta \cos \phi \dot{\theta} - \cos \theta \sin \phi \dot{\phi}), \\
A^{IS} &= -3 \sin 2\theta \cos \phi \dot{r} + 3 (\cos 2\theta \cos \phi \dot{\theta} - \cos \theta \sin \phi \dot{\phi}), \\
A^{FP} &= \sin 2\theta \cos \phi \dot{r}, \\
A^{FS} &= \cos 2\theta \cos \phi \dot{\theta} - \cos \theta \sin \phi \dot{\phi}.
\end{align*}
\]
Radiation pattern

\[ A^N = 9 \sin 2\theta \cos \phi \hat{r} - 6(\cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}), \]

\[ A^{IP} = 4 \sin 2\theta \cos \phi \hat{r} - 2(\cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}), \]

\[ A^{IS} = -3 \sin 2\theta \cos \phi \hat{r} + 3(\cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}), \]

\[ A^{FP} = \sin 2\theta \cos \phi \hat{r}, \]

\[ A^{FS} = \cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}, \]

Far field P - blue
Far field S - red
What rotations do we expect?

- $v_p = 5.2 \text{ km/s}$
- $v_s = 3.0 \text{ km/s}$
- $\rho = 2.6 \text{ g/cm}^3$
Velocity seismograms
M6.5 point source

Total Displacement HradialL

![Graph showing total displacement over time with specific values for displacement and time periods.]
Rotational seismograms
M6.5 point

Total Curl HThetaL

Time

-2 $10^{-6}$
-4 $10^{-6}$
-6 $10^{-6}$
-8 $10^{-6}$

[Graph showing rotational effects over time]
Peak ground motion: rotation, velocity
Seismograms M6.5 point source
(peak motions)
What rotations do we expect?
Effects of a near-surface low-velocity layer

Ground velocity, Maximum 27.45044.2 cm/s

Ground rotation, Maximum 1.490134.2 mrad/s

Time (sec)
What rotations do we expect?
Fault zone effects

Ground velocity, Maximum 56.22 cm/s

<table>
<thead>
<tr>
<th>X-Comp</th>
<th>Y-Comp</th>
<th>Z-Comp</th>
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<tbody>
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Ground rotation, Maximum 3.41 mrad/s

<table>
<thead>
<tr>
<th>Curl: X</th>
<th>Curl: Y</th>
<th>Curl: Z</th>
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What rotations do we expect?

Finite source effects

Do recordings of rotations help in understanding source processes?
What rotations do we expect?
Finite source effects

Ground velocity

Rotation rate

Black: Source model 1  Red: Source model 2
What rotations do we expect?
Finite source effects

The effects are much more distinct for rotations in a wide angular range.

Red: Source model -1; Green Source model 2

Max. Amplitude

Velocity

Angle

Rotation rate

Angle
Earthquake scenario simulation:
Cologne Basin M5.9, 1992

Simulation - red; Observation - black
Observations of teleseismic events:

Rotation rate vs. Acceleration

Plane S-wave travelling in x-direction

\[ u_y = \sin (kx - \omega t) \] displacement

\[ \text{rot}_z = d/dx u_y = \frac{1}{2} k \cos(.) \] rotation

\[ \Theta = 1/2 \omega k \sin(.) \] rotation rate

\[ a = \omega^2 \sin(.) \] acceleration

... Rotation rate is in phase with acceleration
With conversion factor 1/2c:

\[ \Theta = 1/2c \ a \]
Rotational seismograms
M8.3 Hokkaido, 25 September 2003
(recorded in Wettzell, Germany)

N-component

Seismometer

Vertical component

Ring laser
Rotational seismograms
M8.3 Hokkaido, 25 September 2003
(recorded in Wettzell, Germany)
Let us compare the waveforms (red-acc, bue - rot.rate)
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\[ \Theta = \frac{1}{2} c a \]
Other examples

Day # 226  5:00:00.001  2003  TRANSVERSE ACCELERATION & VERTICAL ROTATION RATE
x 10^{3}

- Tranverse Acceleration, €\alpha$
- Vertical Rotation Rate, €\Omega$

$\alpha / 2\Omega = c \approx 0.73 \text{ km/s}.$

Ground acceleration [cm/s²]

G: Lat 49.1442  Lon 12.8789

More examples

Day #270 11:30:0.001 2003 TRANSVERSE ACCELERATION & VERTICAL ROTATION RATE

\[ x \times 10^3 \]

- Tranverse Acceleration, \( \alpha \)
- Vertical Rotation Rate, \( \Omega \)

\[ \frac{\alpha}{2\Omega} = c \approx 0.70 \text{ km/s} \]

GROUND ACCELERATION [cm/s²]

TIME [hr]

Source location: Lat 50.012 Lon 67.824.
More examples

Day # 187  00:00:00.001  2003  TRANSVERSE ACCELERATION & VERTICAL ROTATION RATE

\( \times 10^{-4} \)

\[ \alpha \]

\[ \text{Vertical Rotation Rate, } \vartheta \]

\( \alpha / 2 \vartheta = c \approx 0.71 \text{ km/s} \)

GROUND ACCELERATION [m/s/s]

Global rotational seismograms
Global rotational seismograms

Blue (acc.), red (rotation rate)
Conclusions and Outlook

- Observations of rotational motions for teleseismic events are consistent with collocated recordings of translations (amplitudes, phases?)
- Rotational recordings may help constraining the kinematics of earthquake sources

- Data base with seismic events (tele, local, etc)
- Methods to process rotation data
- Study of phenomenological effects (dynamic rupture, heterogeneous media, anisotropy)
- Installation of ring laser in California