Oppolzer terms: A review

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EQUATIONS OF EARTH ROTATION

$$\frac{\overrightarrow{dH}}{dt} + \vec{\omega} \wedge \vec{H} = \vec{L} \qquad \vec{H} = \Pi\vec{\omega} + \vec{h}$$

$$\vec{\omega} \begin{cases} \Omega m_1 \\ \Omega m_2 \\ \Omega(1+m_3) \end{cases}$$

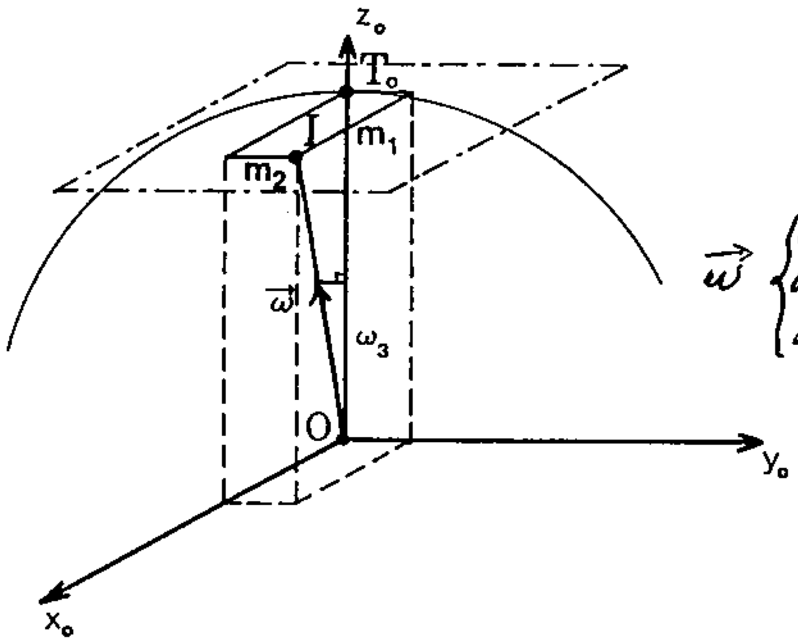
$$\text{Variation of vector } \vec{\omega} : \begin{cases} \textbf{external torque } \vec{L} \\ \textbf{variations of } \mathbf{\Pi} \\ \vec{h} \end{cases}$$

Exchange of **angular momentum**

. between **core and mantle**

. between solid **Earth** and atmosphere

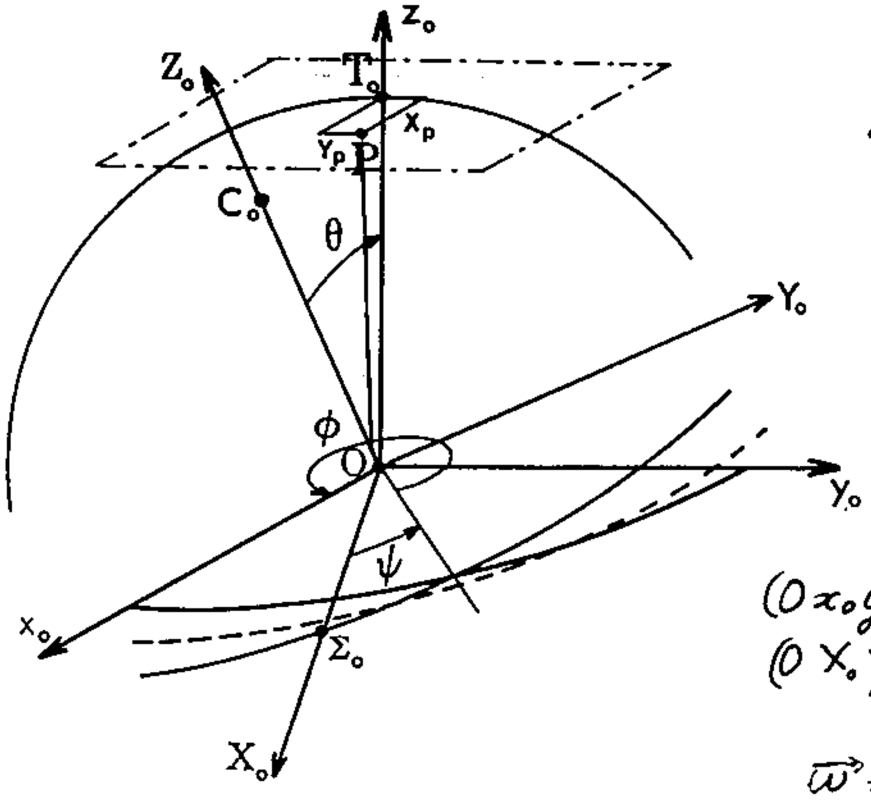
. between solid Earth and ocean



$$\vec{\omega}\begin{cases}\omega_1 = \Omega m_1\\ \omega_2 = \Omega m_2\\ \omega_3 = \Omega(1+m_3)\end{cases}$$

The parameters of the Earth's instantaneous rotation.

Euler's kinematical relations : $\begin{cases}\dot{\theta} + i\dot{\psi}\sin\theta = -\Omega m e^{i\phi}\\ \omega_3 = \dot{\phi} + \dot{\psi}\cos\theta\end{cases}$



$P = CEP$

$(O x_o y_o z_o) = TRS$

$(O X_o Y_o Z_o) = CRS$

$\vec{\omega} = \vec{\omega}(TRS)/(CRS)$

The parameters of the Earth's orientation.

RIGID EARTH (equations)

$$\begin{cases} \psi = \dfrac{c_0}{C-A} + \dfrac{iL}{(C-A)\Omega^2} \\ \psi_3 = \dfrac{1}{C\Omega} \int \mathrm{L}_3 dt \end{cases} \quad \sigma_r = \tfrac{C-A}{A}\Omega$$

$$\begin{cases} \dot{m} - i\sigma_r m = \dfrac{-ic_0}{A}\Omega + \dfrac{L}{A\Omega} \\ \dot{m}_3 = \dfrac{L_3}{C\Omega} \end{cases} \quad (1)$$

Euler's kinematical relations: $\dot{\theta} + i\dot{\psi}\sin\theta = -\Omega m e^{i\Phi}$

$$\left[\frac{d}{dt}(\dot{\theta} + i\dot{\psi}\sin\theta) - i(\tfrac{C}{A}\Omega - \dot{\psi}\cos\theta)(\dot{\theta} + i\dot{\psi}\sin\theta)\right] e^{-i\Phi} = \frac{ic_0}{A}\Omega - \frac{L}{A} \quad (2)$$

where

$$\underbrace{\dot{\theta} + i\dot{\Psi}\sin\theta = -\frac{iL}{C\Omega}e^{i\Phi}}_{\textit{Poisson's equation}} - \frac{iA}{C\Omega}\frac{d}{dt}(\dot{\theta} + i\dot{\Psi}\sin\theta) + \frac{A}{C\Omega}\dot{\Psi}\cos\theta(\dot{\theta} + i\dot{\Psi}\sin\theta)$$

Euler's equations in the Earth **(equation (2) in space, torque $\lambda = -Le^{i\Phi}$)**

$$\begin{cases} A\dot{\omega}_1 + (C-A)\omega_1\omega_3 = L_1 \\ B\dot{\omega}_2 + (C-B)\omega_2\omega_3 = L_2 \\ C\dot{\omega}_3 + (B-A)\omega_1\omega_2 = L_3 \end{cases}$$

Woolard (1953)

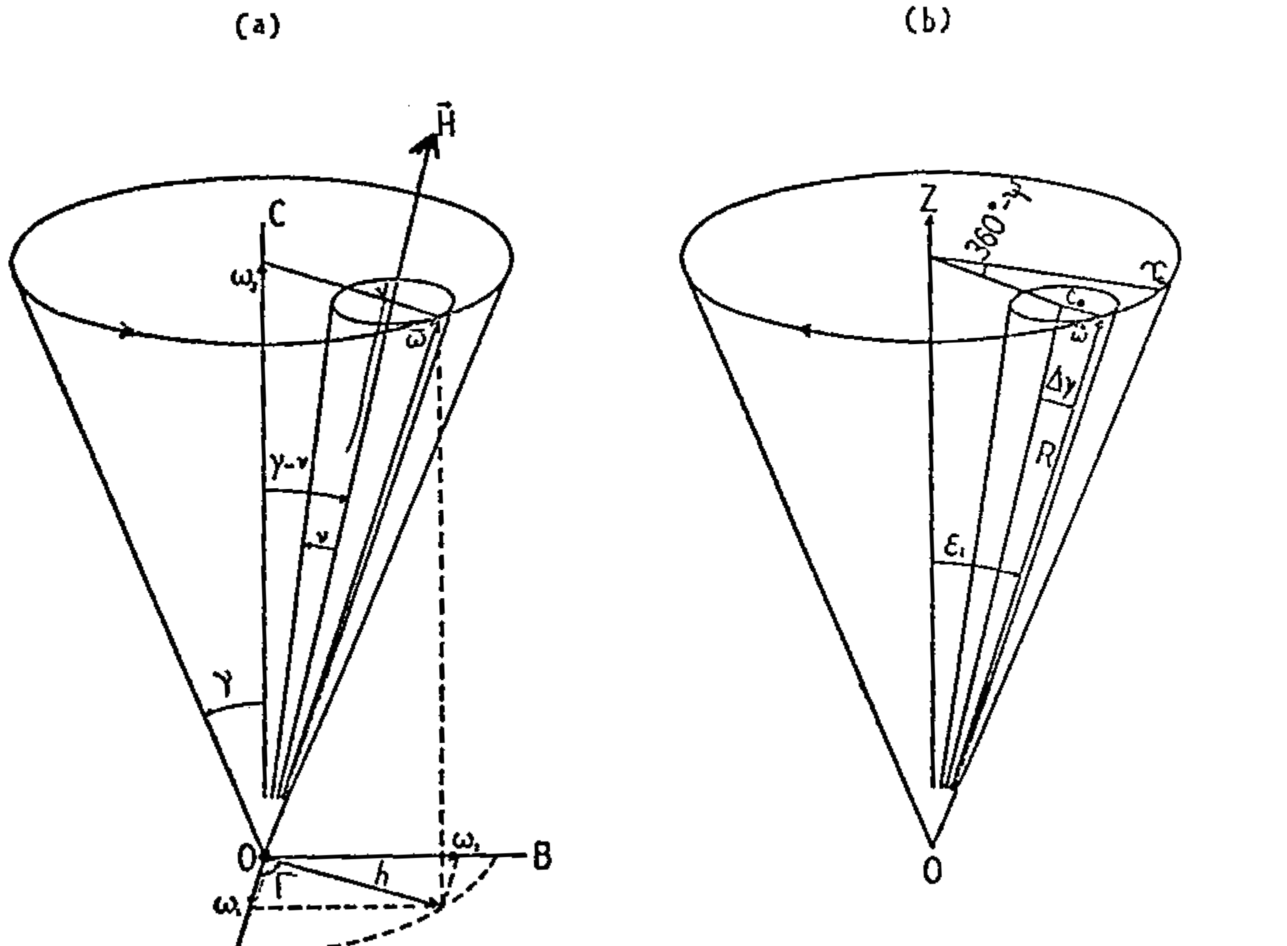


FIGURE 3.—*Poinsot's kinematical representation of the motion of the Earth around the center of mass.*

(a) *The Eulerian motion, in which the large cone rolls on the small cone; the plane through the axes of figure and rotation passes through the angular momentum vector, and rotates counterclockwise around this vector.*

(b) *The lunisolar motion, in which the small cone rolls on the large cone, and progresses clockwise; note that OZ, OC_0, and the axis of rotation do not in general lie in the same plane.*

The relations on the celestial sphere are shown in Figure 2.

ANGULAR DIFFERENCE BETWEEN THE INSTANTANEOUS POLE OF ROTATION AND THE CELESTIAL EPHEMERIS POLE

In the Earth: TRS

"Diurnal nutation": 1 diurnal term ($0''.0087$)
(retrograde) + nearly diurnal term ($\leq 0''.0065$)

⇕ sideral rotation of the Earth (prograde)

In Space: CRS

"Oppolzer's" term: 1 constant term ($0''.0087$)
+ long period terms
(18.6 y, 1 y, 182 d, 13.7 d, ...)

Woolard (1953)

Oppolzer terms for the axis of angular momentum ⇓ axis of figure

Rigid Earth

(CRS)

$$\delta\psi = +\frac{A}{C}\frac{\gamma_0}{\sin\theta}\cos(\varphi+\Gamma_0)$$

~~+0″.000 43T~~

sin	l	l'	F	D	Ω	
+0″.01615	0	0	+2	0	+2	
− 338	0	0	0	0	+1	
+ 334	0	0	+2	0	+1	
+ 309	+1	0	+2	0	+2	
+ 12	−1	0	+2	+2	+1	
+ 10	0	0	+2	+2	+1	
− 28	+1	0	0	0	+1	
+ 64	+1	0	+2	0	+1	
− 46	−1	0	+2	0	+2	
+ 59	−1	0	+2	+2	+2	
+ 49	0	0	+2	+2	+2	
− 12	+1	0	+2	−2	+2	
+ 12	+1	0	+2	+2	+2	
+ 41	+2	0	+2	0	+2	
− 28	−1	0	0	0	+1	
+ 753	0	0	+2	−2	+2	from Sun
+ 44	0	+1	+2	−2	+2	from Sun.

$$\delta\theta = -\frac{A}{C}\gamma_0\sin(\varphi+\Gamma_0) \qquad (54)$$

−0″.008 68

cos	l	l'	F	D	Ω		
+0″.00590	0	0	+2	0	+2		2Ω
+ 113	+1	0	+2	0	+2		3Ω
− 100	0	0	0	0	+1		Ω
+ 99	0	0	+2	0	+1		
− 97	+1	0	0	0	0		
− 16	0	0	0	+2	0		
− 18	+1	0	0	−2	0		
+ 19	+1	0	+2	0	+1		
− 17	−1	0	+2	0	+2		
+ 21	−1	0	+2	+2	+2		
+ 18	0	0	+2	+2	+2		
+ 15	+2	0	+2	0	+2		
+ 275	0	0	+2	−2	+2	from Sun	2L
+ 16	0	+1	+2	−2	+2	from Sun	
− 14	0	+1	0	0	0	from Sun,	

where γ_0 is in seconds of arc. The Sun contributes −0″.00276 to the constant term in $\delta\theta$.

Fedorov (1963)

$$\begin{aligned}\Delta\varphi = &+ 0''.0066 \sin \underline{S} - 0''.0051 \sin (\underline{S} - 2☾) + \\ &+ 0''.0022 \sin (\underline{S} - 2L) + 0''.0010 \sin (\underline{S} - 2☾ - ☊) + \\ &+ 0''.0010 \sin (\underline{S} - 3☾ + \Gamma') - 0''.0009 \sin (\underline{S} - ☊).\end{aligned} \qquad (1.41)$$

Elastic Earth

(TRS)

TABLE 1

Argument	$N_i \sin \theta$	M_i	$\dfrac{\mu_i}{n_1 - \mu_i + (1-\varkappa)\, a^2 n}$	q_i
☊	+6''.8586	+9''.2100	−0.00015	−0.0012
2L	+0 .5066	+0 .5522	+0.00548	+0.0029
2☾	+0 .0811	+0 .0884	+0.07869	+0.0067
2☾ − ☊	+0 .0136	+0 .0183	+0.07884	+0.0013
3☾ − Γ'	+0 .0104	+0 .0113	+0.12246	+0.0013

We find in this way a combination of small variations of latitude of period near to a sidereal day. Oppolzer first pointed out that these appear as a necessary consequence of the motion of the axis of rotation in space, and he also found their expression for a perfectly rigid Earth.

The elastic deformation of the Earth leads to the same relative diminution of the coefficients of all the Oppolzer terms, inasmuch as in $\Delta\varphi$ the factor $1 - \varkappa_0$ appears throughout. This and the lengthening of the period of the free nutation are the only manifestations of the influence of the deformation of the Earth on its rotational motion, inasmuch as the deformation has practically no effect on the motion of the angular momentum in space.

S : sidereal time

PRECESSION AND NUTATION OF THE EQUATOR

Choice of the z-axis

axis of angular momentum : **Woolard 1953, Fedorov 1963, Kinoshita 1977, Souchay et al. 1999)**

axis of instantaneous rotation : **Woolard 1953**

Tisserand mean axis (CEP) : **IAU 1980 series; Bretagnon** *et al.* **1997(CIP): IAU 2000**

H (angular momentum): **(equations of Earth rotation = equations of angular momentum balance)**

- *solutions of Poisson equations* (Woolard 1953)
- *independent of all properties of the Earth other than its moment of inertia (Fedorov 1963)*
- *related to Andoyer variables that are suitable canonical variables*

transformation from (instantaneous axis of rotation)

transformation from angular momentum **to the axis of the ITRS:** Oppolzer terms in the CRS "diurnal nutation" in the TRS

"forced diurnal polar motion",

"diurnal variation of latitude"

wrt axis of figure (Tisserand mean axis = CEP, CIP)

THE CELESTIAL POLE OF REFERENCE

1886 - 1984 : Instantaneous Pole of rotation (Oppolzer 1886, Woolard, 1953)

Pole of figure (Jeffreys, Fedorov 1963, Atkinson 1973, 1975)

1976 : IAU Recommandation: « Atkinson's Pole »

= Instantaneous Pole of rotation + « Oppolzer's » terms

The corresponding concept *was not well understood*

1977 : Resolution Symposium IAU78 : Instantaneous Pole of rotation

1979 : Discussions: Atkinson's pole (Murray 1979, Kinoshita *et al.* 1979) :

1979 : IAU Recommandation *Celestial Ephemeris Pole* (cf. « Atkinson's » Pole)

⇒ IAU 1980 Theory of nutation : CEP (Wahr 1981, Seidelmann 1982)

DEFINITION OF THE CELESTIAL EPHEMERIS POLE

1980 IAU Theory of Nutation (Seidelmann, 1982)

CONCEPT

The CEP is a pole that

- *"has no nearly-diurnal motion with respect to a space-fixed coordinate system or an Earth-fixed coordinate system"*

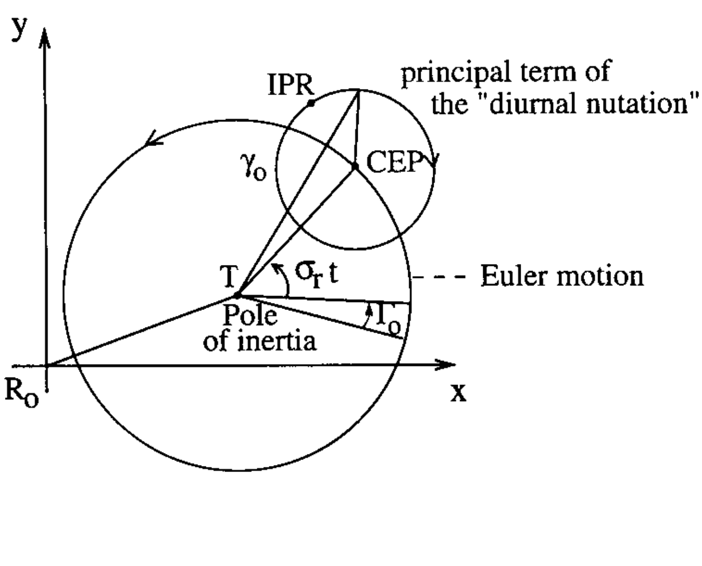
- corresponds to "the axis of figure for the mean surface of a model Earth in which the free motion has zero amplitude" : B axis (Wahr 1979)

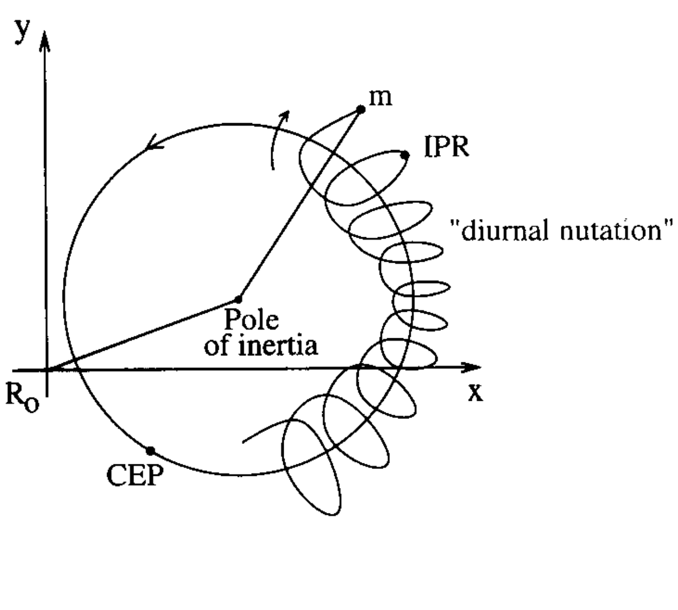
REALIZATION

The 1980 IAU Theory of Nutation *"represents the motion of the CEP with respect to the mean celestial pole of date"*

The CEP is closer to the actually "observed pole" than the instantaneous pole of rotation

CEP AND INSTANTANEOUS POLE ROTATION (IPR)





Changes from the IAU-1980 nutation

- Estimation of the celestial pole offsets
- Improvements of the observations (precision, time resolution, strategy)
- Improvements in the theory (μas accuracy):
  - diurnal and sub-diurnal nutations in the CRS
    - φ, 2φ in the CRS ⇨ 1/3 d, 1/4 d in the TRS
  - diurnal and sub-diurnal pole motion

Realization of the CIP

(IAU Resolution B1.7 of the XXIVthIAU GA, 2000)

- motion of the CIP in the GCRS realized by the best precession-nutation series (periods > 2 days) plus the celestial pole offsets

- motion of the CIP in the ITRS provided by observations taking into account a predictable part specified by a model including high frequency variations

- the corrections to the model for the motion of the CIP in the ITRF may be estimated by extracting the high frequency signal in the pole coordinates
  (1) together with the long periodic motion (Mathews 1999)
  (2) in a second step

Resolution B1.7: Definition of the CIP

(with frequency σ, such that : $\sigma_{TRS} = \sigma_{CRS} - 1$) :

frequency in the TRS __|________|________|________|________|________|________|__

-3.5 -2.5 -1.5 -0.5 +0.5 +1.5 (cpsd)

------ Polar motion | | Polar motion ------

|nutation|

frequency in the CRS __|________|________|________|________|________|________|__

-2.5 -1.5 -0.5 +0.5 +1.5 +2.5 (cpsd)

Table 5.3a: IAU 2000A luni-solar nutations

(Mathews et al. 2002)

	L	Lm	F	D	Om	Period		In Phase				Out of phase		
*						(days)	Psi	dPsi/dt	Eps	dEps/dt	Psi	dPsi/dt	Eps	Deps/dt
*							(mas)	(mas/c)	(mas)	(mas/c)	(mas)	(mas/c)	(mas)	(mas/c)
	0	0	0	0	1	-6798.383	-17206.4161	-17.4666	9205.2331	0.9086	3.3386	0.0029	1.5377	0.0002
	0	0	2	-2	2	182.621	-1317.0906	-0.1675	573.0336	-0.3015	-1.3696	0.0012	-0.4587	-0.0003
	0	0	2	0	2	13.661	-227.6413	-0.0234	97.8459	-0.0485	0.2796	0.0002	0.1374	-0.0001
	0	0	0	0	2	-3399.192	207.4554	0.0207	-89.7492	0.0470	-0.0698	0.0000	-0.0291	0.0000
	0	1	0	0	0	365.260	147.5877	-0.3633	7.3871	-0.0184	1.1817	-0.0015	-0.1924	0.0005
	0	1	2	-2	2	121.749	-51.6821	0.1226	22.4386	-0.0677	-0.0524	0.0002	-0.0174	0.0000
	1	0	0	0	0	27.555	71.1159	0.0073	-0.6750	0.0000	-0.0872	0.0000	0.0358	0.0000
	0	0	2	0	1	13.633	-38.7298	-0.0367	20.0728	0.0018	0.0380	0.0001	0.0318	0.0000

.....

OPPOLZER TERMS CORRESPONDING TO IAU 2000A PRECESSION/NUTATION

$dX_{OP} = d\psi \sin\theta$: *103 periodic terms larger than 1 microarcsecond*

	unit: microarcseconds		*l*	*l'*	*F*	*D*	*Ω*
1	7162.55	-10.06	0	0	2	0	2
2	3137.83	2.51	0	0	2	-2	2
3	1472.34	-2.33	0	0	2	0	1
4	1412.76	-4.02	1	0	2	0	2
5	-1354.03	0.22	0	0	0	0	1
6	289.08	-0.86	1	0	2	0	1
7	-267.30	-0.69	1	0	-2	-2	-2
8	231.83	-0.96	0	0	2	2	2
9	196.77	0.01	1	0	-2	0	-2
10	192.99	-0.86	2	0	2	0	2
11	184.30	0.14	0	1	2	-2	2
12	-120.10	0.03	1	0	0	0	1
13	114.52	-0.27	1	0	0	0	-1
14	57.88	-0.34	1	0	2	2	2
15	-54.72	-0.16	1	0	-2	-2	-1
16	-51.53	0.01	1	0	2	-2	2
17	47.30	-0.20	0	0	2	2	1
18	39.87	0.01	1	0	-2	0	-1
19	39.33	-0.18	2	0	2	0	1
20	-38.79	-0.02	0	0	2	-2	1

2 Poisson terms larger than 1 microarcsecond/cy

104	-3.55	0.35	0	0	2	0	2
105	-1.65	0.15	0	0	2	-2	2

RELATIONSHIP BETWEEN EOP AND $\vec{\omega}$

examples

a) Geometrical observations (astrometry, VLBI)

for $m = \gamma e^{\sigma t} \Rightarrow p = (1+\frac{\sigma}{\Omega})^{-1} m = \gamma' e^{\sigma t}$ with $\gamma' = (1+\frac{\sigma}{\Omega}\gamma)$

(1) Chandler : $\frac{\sigma}{\Omega} \simeq 1/430 \Rightarrow \frac{\gamma'}{\gamma} = 0.997$ for $\gamma = 1'' \Rightarrow \gamma' - \gamma = 2mas$

(2) FCN : $\sigma \simeq (-1+\varepsilon)\Omega \Rightarrow \frac{\gamma'}{\gamma} = \frac{1}{\varepsilon} \simeq 430$

(3) oceanic diurnal and semi-diurnal tides

$\Rightarrow$ axis of rotation $\neq$ axis of the CEP according to the tide $\frac{\gamma'}{\gamma} = 1, 2, 3$

b) Dynamical observations (SLR)

same effect as for a) + additional factor $[1 + \dot{\mathcal{N}}((\sigma + \Omega + \dot{\mathcal{N}}]$

$\mathcal{N}$: rate of precession of the node orbit

c) Gravimetric or gyroscopic observations

$\Rightarrow \vec{\omega}$

$\Delta g \rightarrow (1 + \frac{\Lambda a \Omega^2}{g}) m$

SUB-DIURNAL VARIATIONS OF EARTH ROTATION

- Can be accurately related to theoretical quantities for the instantaneous axis of rotation

- Variations up to 3 hours can be provided for $\vec{w}$ (Bolotin *et al.* *1995*) by intensive VLBI observations using the definition of $\vec{w}$ through the matrix transformation

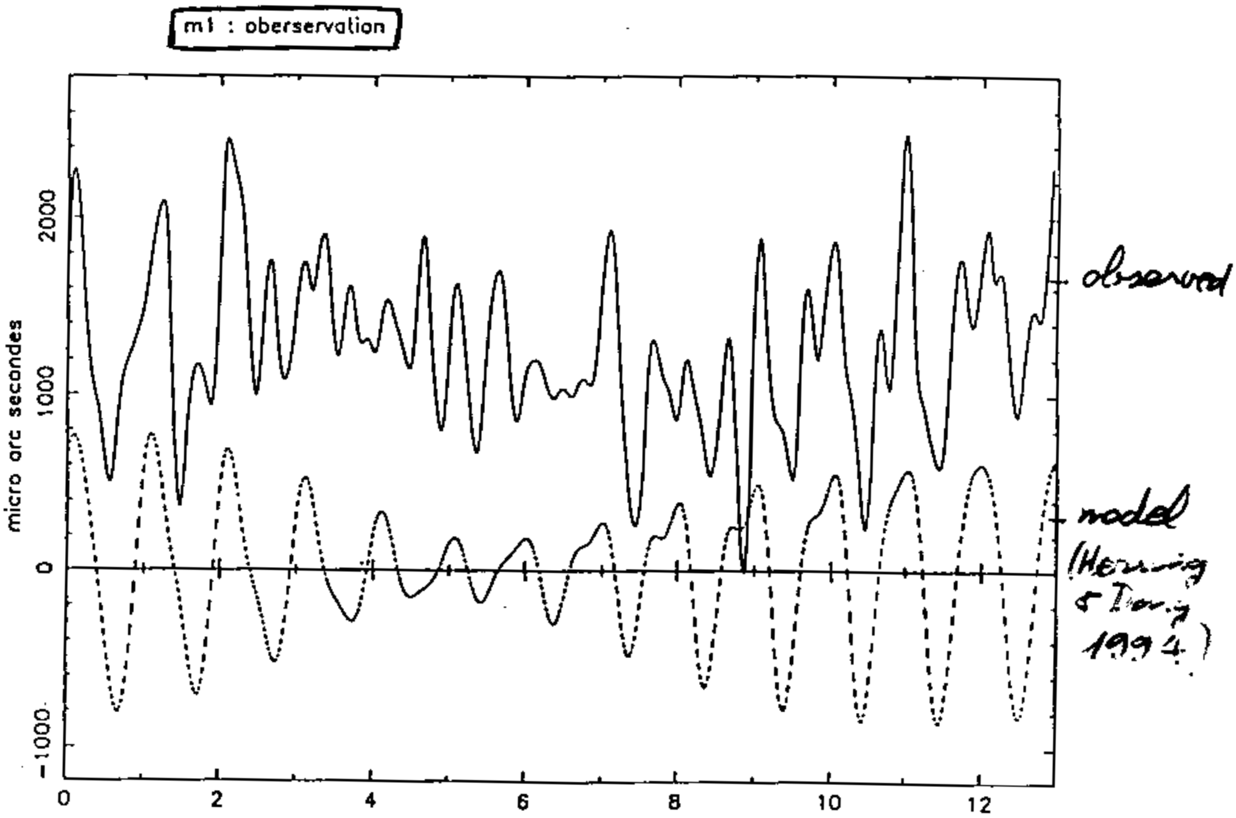
$$[CRS] = Q[TRS] = AQ_{ref}[TRS]$$

with $A = R_1(\alpha_1).R_2(\alpha_2)R_3(\alpha_3)$

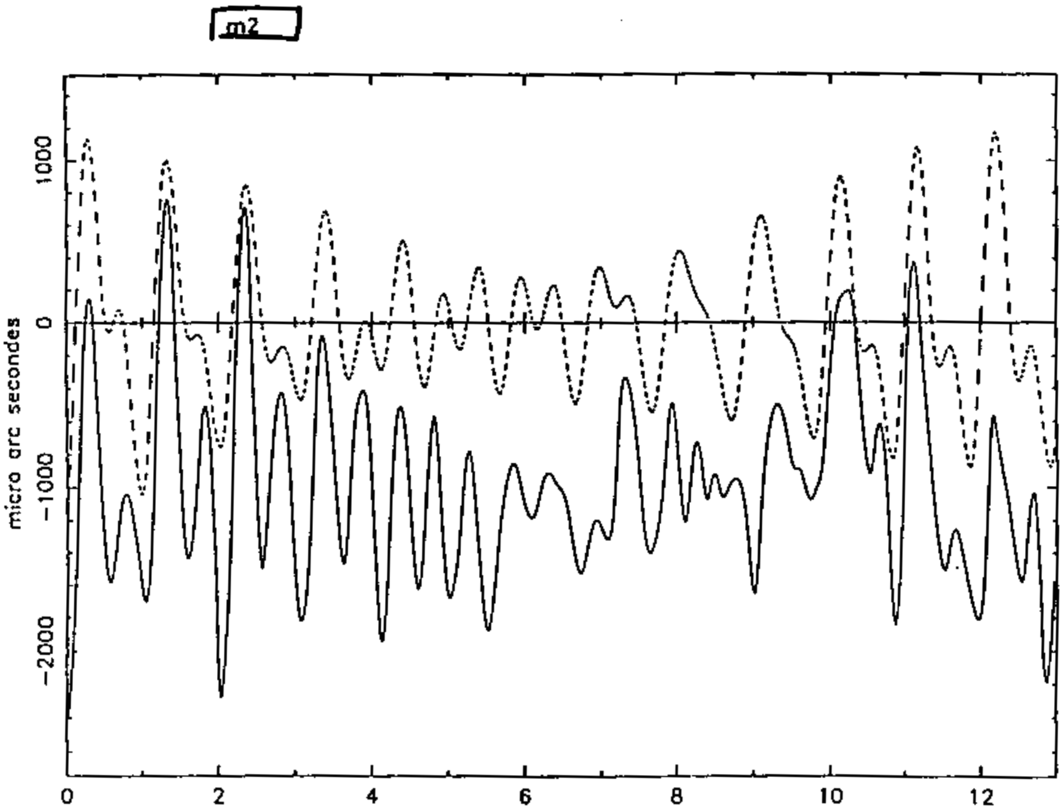
$$d\vec{w})_{TRF} = \begin{pmatrix} d\omega_1 \\ d\omega_2 \\ d\omega_3 \end{pmatrix} \quad \text{such that :}$$

$$\begin{pmatrix} 0 & -\delta\omega_3 & \delta\omega_2 \\ \delta\omega_3 & 0 & -\delta\omega_1 \\ \delta\omega_2 & \delta\omega_1 & 0 \end{pmatrix} = A^{-1} Q_{ref}^{-1} \dot{Q}_{ref} A + A^{-1}\dot{A} - Q_{ref}^{-1}\dot{Q}$$

Bolotin, Bizouard, Loyer, Capitaine (1997)



High frequency motion of the instantaneous rotation axis % TRS



VLBI "intensive" campaign

EARTH ORIENTATION MATRIX AND ROTATION VECTOR

- **Compute orientation matrix at a given time or in real time**
- **Compute time series of the Earth orientation matrix or derived parameters between two dates :**

Time scale for the dates ◉ UTC ○ TAI

First date : year 2004 month 1 day 1 hour 0 min 0 second 0 **Step in secor**
Last date : year 2004 month 3 day 15 hour 0 min 0 second 0

Include combined EOP C04 (1962 - today + 6 months prediction) when available :
celestial pole offsets ☑ UT1 - UTC ☑ Polar motion ☑

Submit request

Include diurnal and semidiurnal variations produced by ocean tides ☑ (IERS conventions 2000)

Reset

○ **Orientation matrix**

○ **Projection of the geographic axis** on the equatorial plane J2000.0 OXY (unit : arcsecond) Draw
N.B.: by taking UT1-UTC and the polar motion to zero one obtains the equatorial celestial coordinates of the Celestial Intermediat

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Components (ω_1,ω_2,ω_3) of the instantaneous rotation vector ω :

◉ **in the International terrestrial frame Oxyz**
○ **in the local frame** Ox'y'z' : Oz'=vertical (either geographic or astronomical), Ox' tangent to the meridian, towards South, Oy' in the horizontal plane, directly orthogonal to Ox'.
Latitude (degree) 50 longitude (degree) 0
○ **in the International celestial frame** OXYZ

○ do not draw
◉ draw (date,ω_1)
○ draw (date,ω_2)
○ draw (date,ω_3)
○ draw (ω_1,ω_2)

○ **in nanorad/s** ◉ **(ω_1/Ω,ω_2/Ω) in arcsec (Ω=7.292115 e-5 rad/s)**

○ **Precession-nutation matrix**

Transformation coordinate M from the international terrestrial reference system (ITRF) to the internationa reference frame (ICRF) : Celestial coordinates (X Y Z) = M x Terrestrial coordinates (x y z) - We use th CODE of the VLBI analysis Software GLORIA developped by Anne-Marie Gontier **(Paris Observatory)**, compatible with the IAU 2000 resolutions

- **from 1962 to the current week** the matrix can include the whole EOP set of the IERS combined s (time resolution for such EOP fluctuations is about 6 days), as well as diurnal and semi-diurnal eff produced by ocean tides, which amount about 0.001". Accuracy is 0.0001", corresponding to 5 10 turn the time resolution of the matrix can reach approximatively 6 hours.
- **from the current week to 6 months after** the matrix is computed from the prediction of the polar UT1-UTC.
- **outside the range [1962, current week+ 6 months prediction]** the rotation matrix is restricted to

NEWS IERS EOP PC

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Earth orientation parameters
Astro-geodetic techniques
Models/Software
Leap second

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EOP combined series
Synoptic of EOP series
Rotation matrix
Interactive EOP
Bulletins B, C, D
Operational EOP series
Long term EOP series

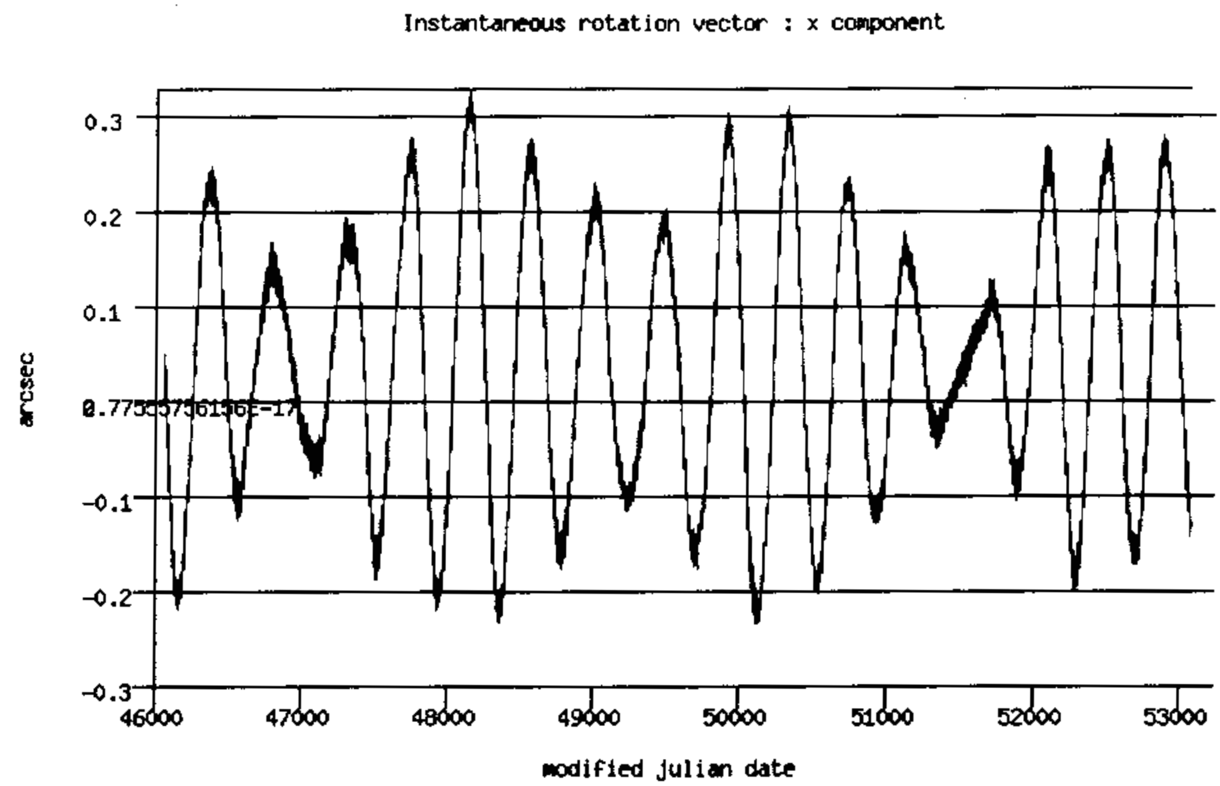
Analysis

Interactive EOP analysis
Geophysical excitation
Consistency

Useful constants

Related sites

WEB realizer:
Christian BIZOUARD


Instantaneous rotation vector : x component
arcsec
0.3
0.2
0.1
-0.1
-0.2
-0.3
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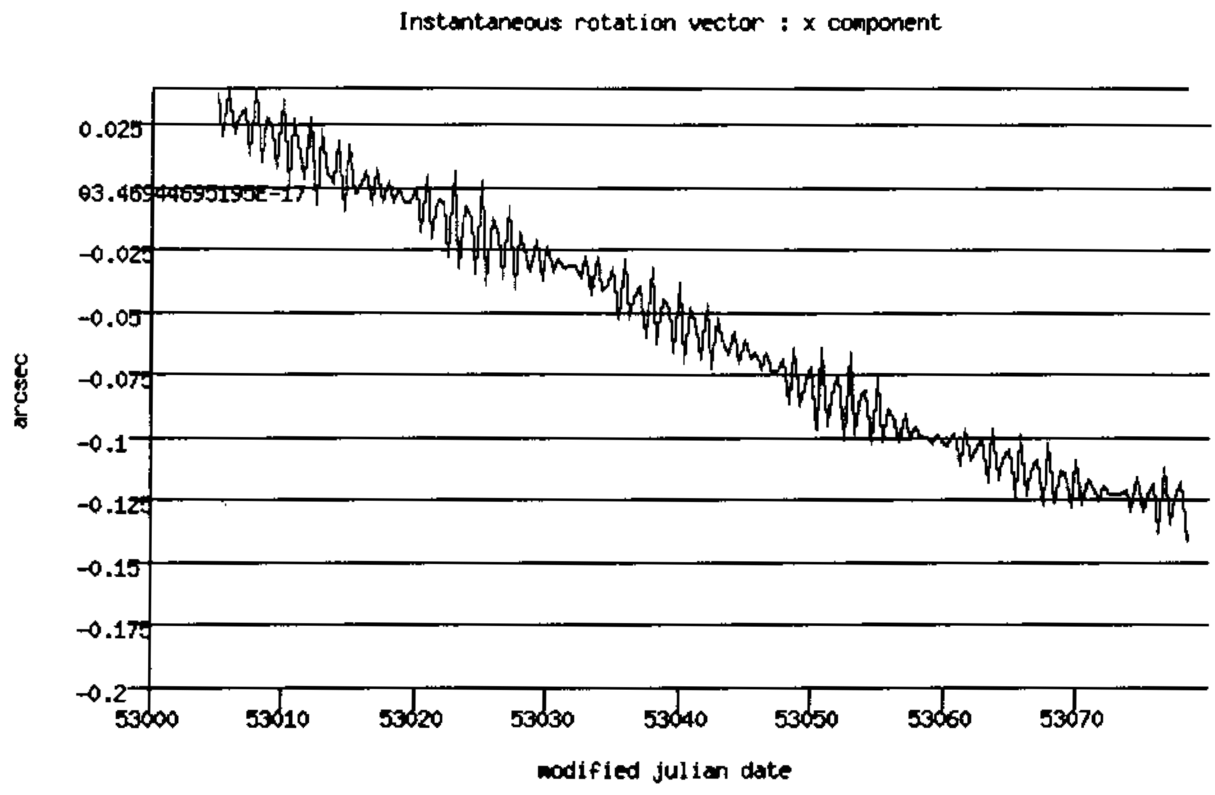
47000
48000
49000
50000
51000
52000
53000
modified julian date


Instantaneous rotation vector : x component
arcsec
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-0.025
-0.05
-0.075
-0.1
-0.125
-0.15
-0.175
-0.2
53000
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53020
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modified julian date